Amplify Math
INDIANA

Algebra 1

UNIT 3 | INDIANA LESSON 10A

Input and Output

FEATURING Amps POWERED BY desmos

Input and Output

Let's determine all possible inputs and outputs for a function.

Focus

Goals

1. Given a description of a function that represents a situation, determine a reasonable set of inputs and outputs.
2. Understand that the set of all possible outputs is dependent on the set of all possible inputs.

Coherence

Today

In this lesson, students focus their attention on possible input and output values. They identify the sets of possible inputs and outputs of functions and describe them using words, lists of numbers, or inequalities (if appropriate). Students' analyses of inputs and outputs continue to be grounded in context, allowing many chances for them to reason quantitatively and abstractly **(MP2)**.

Previously

In Lesson 8, students considered (even if only peripherally) input and output values that would make sense in the context of a discrete function.

Coming Soon

In Lesson 11, students will formally define the terms *domain* and *range* and relate them to the features of a graph.

Rigor

- Students develop **conceptual understanding** of the restrictions on certain functions, and how the outputs depend on the inputs.

Standards

Addressing

AI.F.3
Identify the domain and range of relations represented in tables, graphs, **verbal descriptions, and equations.**

Pacing Guide

Suggested Total Lesson Time **~60 min**

Warm-up	Activity 1	Activity 2	Summary	Exit Ticket
10 min	20 min	20 min	5 min	5 min
Pairs	Small Groups	Small Groups	Whole Class	Independent
MP2	MP2	MP2		
AI.F.3	AI.F.3	AI.F.3	AI.F.3	AI.F.3

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, pre-cut cards, one set per group
- Activity 1 PDF, *Function Inputs and Outputs* (for display)

Math Language Development

Review words

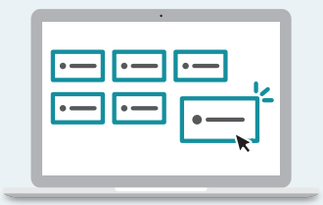
- *input*
- *output*

Amps | **Featured Activity**

Activity 1

Digital Card Sort

This activity benefits from the digital card sort by being both easy on materials preparation and helping students keep their materials organized.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students may feel confused when sorting the *possible* and *impossible* value cards in Activity 1 **(MP8)**. Ask students whether they have any organizational strategies they like to use when needing to sort objects into different groups.

Modifications to Pacing

You may want to consider this additional modification if you are short on time.

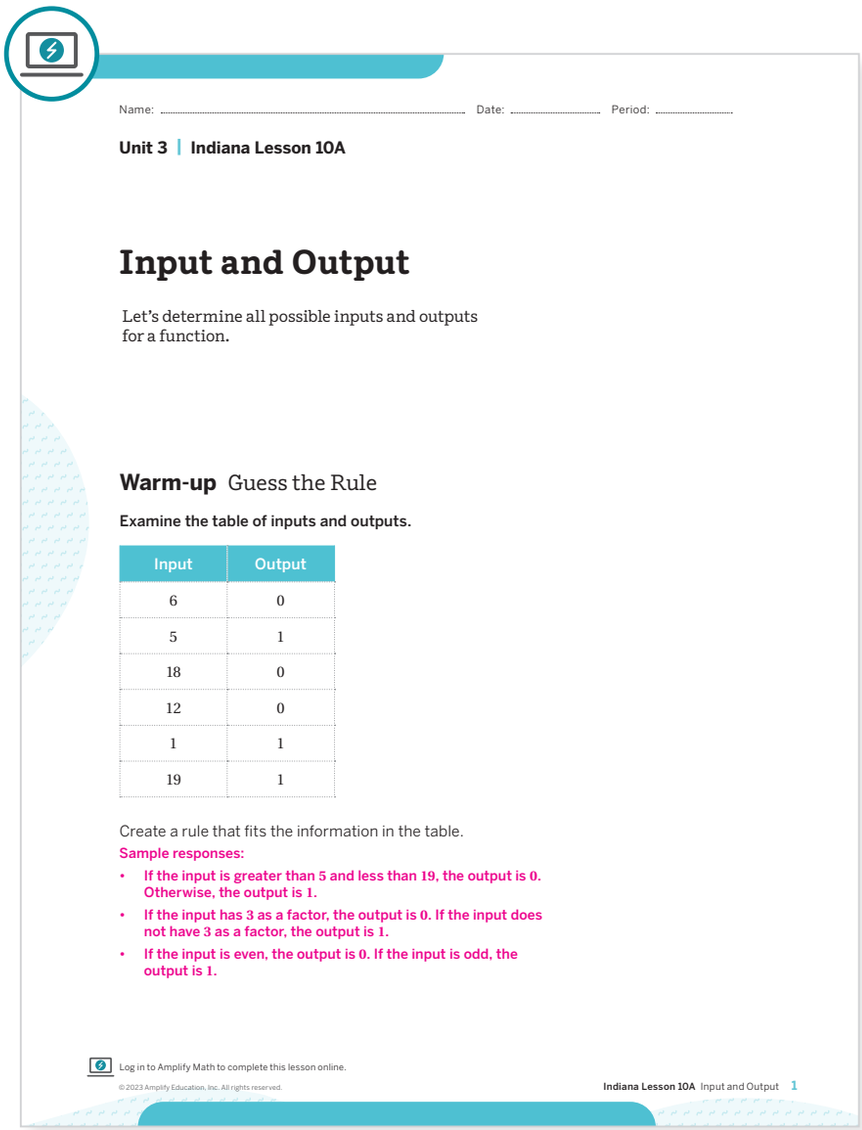
- In **Activities 1 and 2**, have students only complete the first three problems.

Pairs | 10 min

Warm-up Guess the Rule

MP2
AI.F.3

Students examine a table of inputs and outputs in order to guess the rule and reason about which inputs and outputs are possible.



Name: ..... Date: ..... Period: .....

Unit 3 | Indiana Lesson 10A

Input and Output

Let's determine all possible inputs and outputs for a function.

Warm-up Guess the Rule

Examine the table of inputs and outputs.

Input	Output
6	0
5	1
18	0
12	0
1	1
19	1

Create a rule that fits the information in the table.

Sample responses:

- If the input is greater than 5 and less than 19, the output is 0. Otherwise, the output is 1.
- If the input has 3 as a factor, the output is 0. If the input does not have 3 as a factor, the output is 1.
- If the input is even, the output is 0. If the input is odd, the output is 1.

Log in to Amplify Math to complete this lesson online.

1 Launch

Review the definition of a *function* — a rule that assigns exactly one output to each possible input. Say, "I used a rule to create this table. Can you guess my rule?"

2 Monitor

Help students get started by asking, "What do the rows with the same outputs have in common?"

Look for points of confusion:

- **Thinking that the relationship is not a function because some outputs are the same.** Have students visually map the inputs to the outputs to check whether the same input has different outputs.
- **Creating a rule that only identifies either the inputs or the outputs.** Remind students that a function is a type of rule that assigns an output according to the input and the rule.

Look for productive strategies:

- Creating multiple rules that work for the given function **(MP2)**.

3 Connect

Display the table from the Warm-up.

Have pairs of students share their rules with the class. Be sure to record, for all the class to see, any language about sets or groups of numbers that comes up during the share. Say, "The rule I used to create this table is: 'Even inputs give an output of 0. Odd inputs give an output of 1.'"

Ask:

- "What are some other possible inputs?"
- "What are the only possible outputs?"
- "Where else in this unit have you seen that there are only certain numbers that might be used for certain functions?"

Highlight that the rule for a function can be simple, complex, or something in between. Let students know that, in the other activities, they will be considering functions that are a bit more complex than the one in the Warm-up.

Differentiated Support

Accessibility: *Clarify Vocabulary and Symbols*

Display review vocabulary terms for sets of numbers, such as *rational, integer, even, odd,* and *whole*. This will provide support for students both during the Warm-up and throughout the lesson.

Extension: *Math Enrichment*

There are many possible rules that can fit the table. Encourage students to determine and describe as many of them as they can.

Power-up

To power up students' ability to use inequalities to describe possible values, ask:

Complete each statement to make it true for the given set of values.

1. **Values:** 3, 4, 6, 2.
   These values are all greater than or equal to ___ and less than or equal to ___.
2. **Values:** 2.5, $1\frac{1}{4}$, −3, $2\frac{3}{4}$
   These values are all greater than ___ and less than ___.

Use: Before Activity 1

Informed by: Performance on Lesson 10, Practice Problem 6

Small Groups | 20 min

Activity 1 Card Sort: Possible or Impossible?

MP2
AI.F.3

Students classify numerical values as possible or impossible inputs for each function based on the situation represented by each function.

Amps Featured Activity Digital Card Sort

Activity 1 Card Sort: Possible or Impossible?

You will be given a set of cards that each contain a number. Decide whether each number is a possible input for the functions described here. Sort the cards into two groups — possible inputs and impossible inputs. Record your sorting decisions.

1. The area of a square, in square centimeters, is a function of its side length, s, in centimeters. The equation $A(s) = s^2$ defines this function.

a. Possible inputs:
$9, \frac{3}{5}, 15, 0.8, 4, \frac{25}{4}, 0.001, 6.8, 72$

b. Impossible inputs:
$-3, 0, -18$

2. The relationship between temperature in Celsius and the temperature in Fahrenheit can be represented by a function F. The equation $F(c) = 1.8c + 32$ defines this function, where c is the temperature in Celsius and $F(c)$ is the temperature in Fahrenheit.

a. Possible inputs:
$-3, 9, \frac{3}{5}, 15, 0.8, 4, 0, \frac{25}{4}, 0.001, -18, 6.8, 72$

b. Impossible inputs:
None

2 Unit 3 Functions and Their Graphs

1 Launch

Activate background knowledge by asking, "If someone asks you to guess a number from 1–10, what are some possible responses? What are some responses that wouldn't make sense?" Distribute cards from the Activity 1 PDF to each group. Consider asking groups to pause after sorting possible inputs for the first function and to discuss their decisions with another group.

2 Monitor

Help students get started by suggesting they begin by considering each card one at a time. For Problem 1, hold up a card and ask, "Could this be the side length of a square, in centimeters? Why or why not?"

Look for points of confusion:

- **Conflating "not typical" with "not possible."** Some students may think $\frac{25}{4}$ is not possible as an amount of centimeters because they are usually represented as decimals. You might say, "That may be true, but is it *possible* to have $\frac{25}{4}$ centimeters?"
- **Thinking 72 is a possible input for Problem 3.** Ask, "Is it possible to play the game for 72 hours, according to the situation?"

Look for productive strategies:

- Generalizing the possible input values to sets of numbers.
- Using an inequality to represent the set of possible inputs.

Activity 1 continued >

Differentiated Support

Accessibility: *Vary Demands to Optimize Challenge*

If students need more processing time, have them focus on Problems 1–3 and, if they have time available, work on Problem 4. The function in Problem 4 will be revisited during the Summary.

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which the digital card sort allows students to quickly and easily organize their cards.

Math Language Development

MLR2: Collect and Display

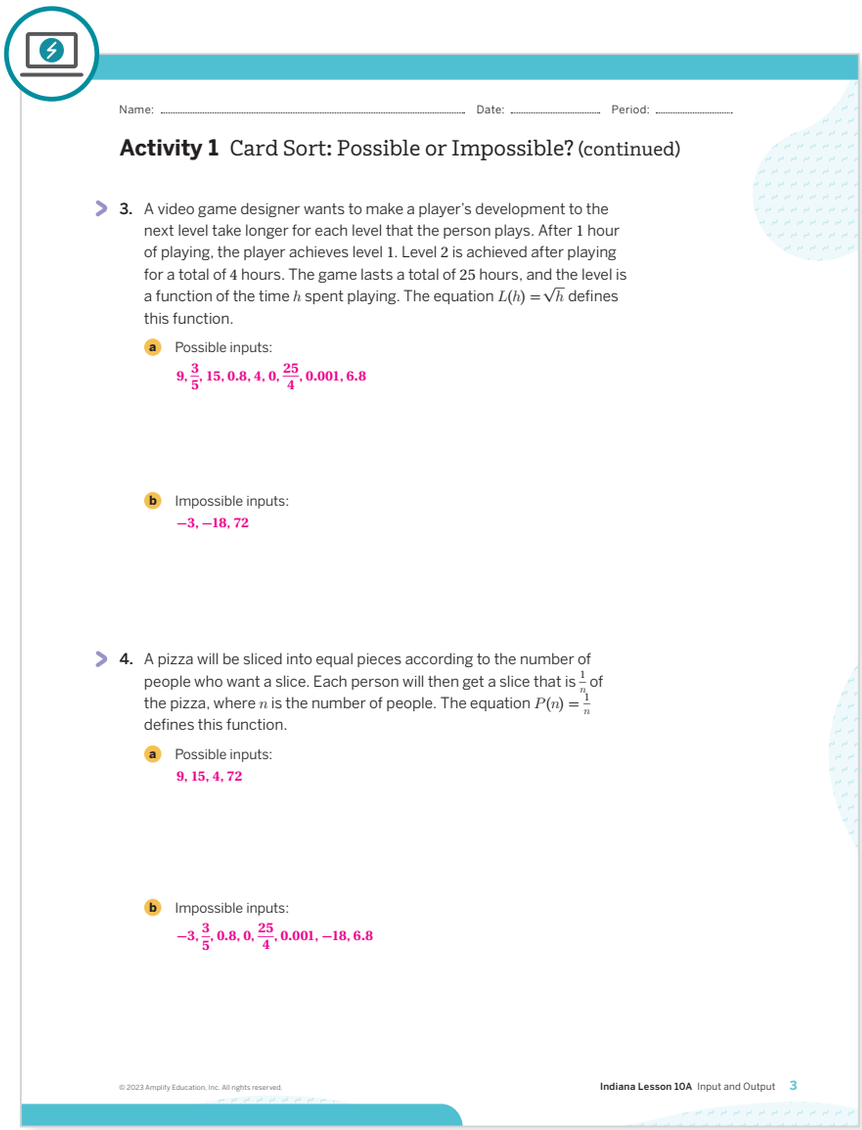
During the Connect, collect informal student language used to describe possible inputs. As you progress through the lesson, highlight connections between informal descriptions, formal vocabulary for sets of numbers, and inequality statements that represent the set of possible inputs.

English Learners

Encourage students to use language from the class display to support developing mathematical language in this unit.

Activity 1 Card Sort: Possible or Impossible? (continued)

MP2
AI.F.3



Name: ..... Date: ..... Period: .....

Activity 1 Card Sort: Possible or Impossible? (continued)

3. A video game designer wants to make a player's development to the next level take longer for each level that the person plays. After 1 hour of playing, the player achieves level 1. Level 2 is achieved after playing for a total of 4 hours. The game lasts a total of 25 hours, and the level is a function of the time h spent playing. The equation $L(h) = \sqrt{h}$ defines this function.

a. Possible inputs:
$9, \frac{3}{5}, 15, 0.8, 4, 0, \frac{25}{4}, 0.001, 6.8$

b. Impossible inputs:
$-3, -18, 72$

4. A pizza will be sliced into equal pieces according to the number of people who want a slice. Each person will then get a slice that is $\frac{1}{n}$ of the pizza, where n is the number of people. The equation $P(n) = \frac{1}{n}$ defines this function.

a. Possible inputs:
$9, 15, 4, 72$

b. Impossible inputs:
$-3, \frac{3}{5}, 0.8, 0, \frac{25}{4}, 0.001, -18, 6.8$

3 Connect

Display the Activity 1 PDF, *Function Inputs and Outputs*.

Have groups of students share their sorting results. Record and display for the class to see the values students considered possible and impossible inputs for each function. Discuss any remaining disagreements students might have about particular values.

Ask, "What are some common sets of numbers that may be impossible for certain functions?" **(MP2)** Sometimes negative numbers or non-whole numbers do not work as possible inputs for some functions.

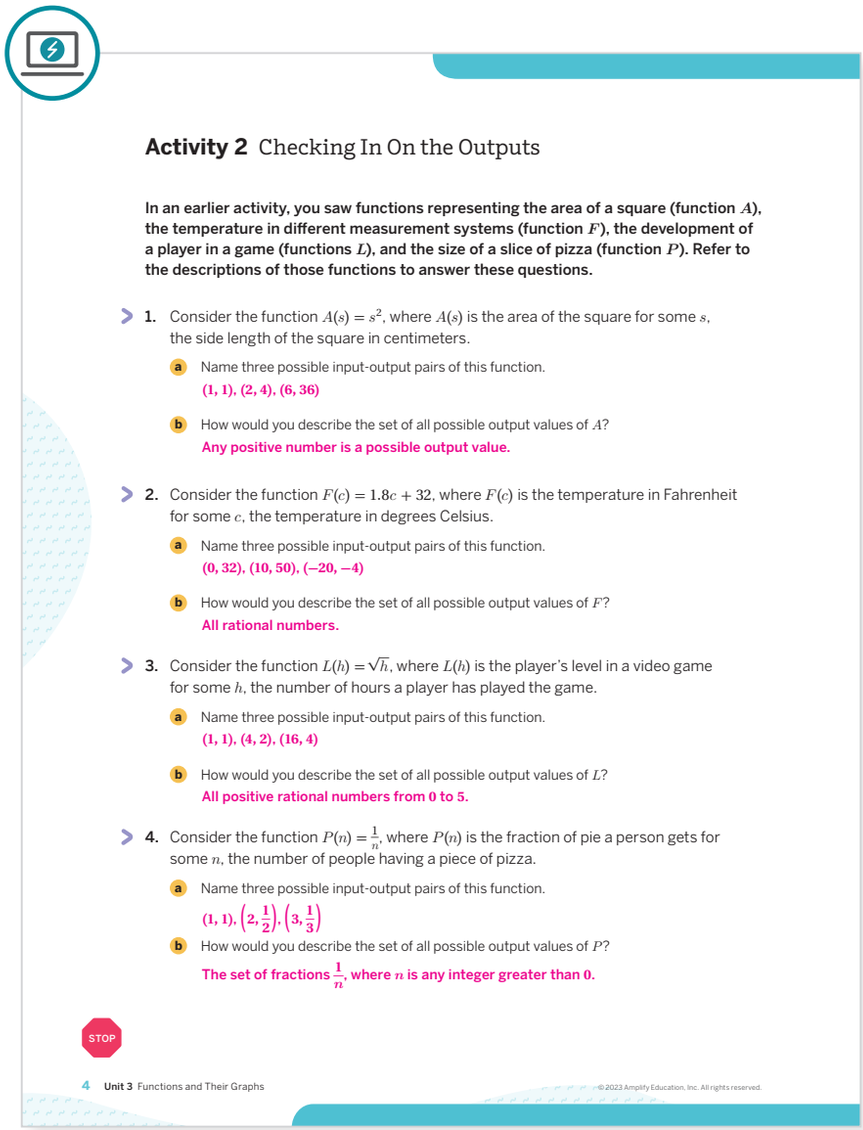
Highlight that each function has its own set of possible and impossible values. For some functions, all values may be possible. It can be useful to consider special sets of numbers as whole groups when considering which inputs are possible or impossible. 0 is another important value to consider.

Small Groups | 20 min

Activity 2 Checking In On the Outputs

MP2
AI.F.3

Students revisit the functions from Activity 1 and consider how the possible inputs affect the set of possible outputs. They also see how a graph of a function can help with this.



Activity 2 Checking In On the Outputs

In an earlier activity, you saw functions representing the area of a square (function A), the temperature in different measurement systems (function F), the development of a player in a game (functions L), and the size of a slice of pizza (function P). Refer to the descriptions of those functions to answer these questions.

1. Consider the function $A(s) = s^2$, where $A(s)$ is the area of the square for some s, the side length of the square in centimeters.

a. Name three possible input-output pairs of this function.
(1, 1), (2, 4), (6, 36)

b. How would you describe the set of all possible output values of A?
Any positive number is a possible output value.

2. Consider the function $F(c) = 1.8c + 32$, where $F(c)$ is the temperature in Fahrenheit for some c, the temperature in degrees Celsius.

a. Name three possible input-output pairs of this function.
(0, 32), (10, 50), (−20, −4)

b. How would you describe the set of all possible output values of F?
All rational numbers.

3. Consider the function $L(h) = \sqrt{h}$, where $L(h)$ is the player's level in a video game for some h, the number of hours a player has played the game.

a. Name three possible input-output pairs of this function.
(1, 1), (4, 2), (16, 4)

b. How would you describe the set of all possible output values of L?
All positive rational numbers from 0 to 5.

4. Consider the function $P(n) = \frac{1}{n}$, where $P(n)$ is the fraction of pie a person gets for some n, the number of people having a piece of pizza.

a. Name three possible input-output pairs of this function.
$(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right)$

b. How would you describe the set of all possible output values of P?
The set of fractions $\frac{1}{n}$, where n is any integer greater than 0.

STOP

4 Unit 3 Functions and Their Graphs

1 Launch

Keep students in small groups. Give them a few minutes of quiet work time, and then a moment to share their responses with their group. Leave a few minutes for a whole-class discussion.

2 Monitor

Help students get started by asking, "How can you determine the output for one of the inputs?"

Look for points of confusion:

- **Thinking they need to methodically substitute every input into the function.** Ask, "Now that you have tried a couple of inputs, can you predict what types of numbers other outputs must be?".

Look for productive strategies:

- Organizing inputs and outputs in a table.
- Using an inequality to represent the set of possible outputs. **(MP2)**

3 Connect

Display the Activity 1 PDF, *Function Inputs and Outputs*. Add to the output section for each function as groups of students share.

Have groups of students share their descriptions of the possible outputs for each function.

Highlight how the sets of possible inputs and outputs can often be described by either naming a set of numbers (i.e., "integers," "whole numbers") or by using an inequality to represent the possible values.

Ask, "How can the *description* of a function change the set of possible inputs and outputs, compared to just seeing it in function notation?" The description could make it discrete or continuous. For example, when the output value describes an amount of people, the function is usually considered discrete.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Ask students to recall how coordinate pairs can be used to describe independent and dependent variable pairs. Ask, "In a function, which variable is independent and which is dependent?"

Math Language Development

MLR2: Collect and Display

During the Connect, collect informal student language used to describe possible outputs. Add these to the display from Activity 1.

English Learners

Encourage students to use language from the class display to support developing mathematical language in this unit.

Whole Class | 5 min

Summary

AI.F.3

Review and synthesize how to determine the set of possible inputs and outputs for a given function.

Name: ..... Date: ..... Period: .....

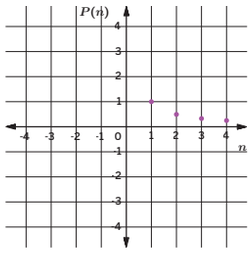
Summary

In today's lesson . . .

You saw that for certain functions, there are a limited number of inputs and outputs that are possible. Once you know all the possible inputs, you can determine the outputs that make sense for the situation. The graph of a function can help to identify possible and impossible input and output values.

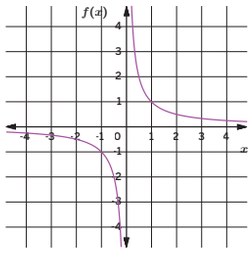
Sometimes, functions that appear similar can have different sets of possible inputs and outputs because they have different contexts.

$P(n) = \frac{1}{n}$, where n is a number of people



Possible inputs	Possible outputs
All positive natural numbers.	All fractions with 1 in the numerator and natural numbers, except 0, in the denominator.

$f(x) = \frac{1}{x}$, where x is any rational number except 0.



Possible inputs	Possible outputs
All rational numbers, except 0.	All rational numbers, except 0.

Reflection:

Synthesize

Display Activity 1 PDF, *Function Inputs and Outputs*.

Have students share how seeing the graph of a function can help them to reason about possible inputs and outputs for a function.

Ask:

- "Why does it make sense to check which inputs are possible before checking which outputs are possible?" The outputs are dependent upon the inputs, so I can really only know which outputs are possible once I have defined the possible inputs.
- "In what ways is the graph of $P(n)$ different from the graph of $f(x)$? Why might that be? How does the definition of the input variable affect the look of the graph?" Because n is defined as an amount of people, it does not make sense to have a negative or fractional amount of people, so there are no negative or fractional values of x plotted.
- "Why might it be useful to check for 0? Are there any other special cases that you should check for?"
- "What are some functions where the input can never be 0?"

Highlight that, in the next lesson, students will see that the sets of possible inputs and outputs are so important that each actually has its own special name.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection. Encourage them to record any notes in the *Reflection* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "Why should functions be analyzed graphically?"

Independent | 5 min

Exit Ticket

AI.F.3

Students demonstrate their understanding of possible input and output values by reasoning about sets of numbers that are possible for a given function.

Printable

Name: _____ Date: _____ Period: _____

Exit Ticket 3.10A

Function q gives the number of minutes a person sleeps as a function of the number of hours they sleep in a 24-hour period.

Mark the columns with a "yes" or "no" to indicate whether each value is a possible input or output of this function.

	Possible input?	Possible output?
negative value	no	no
0	yes	yes
value less than 1	yes	yes
24	yes	yes
25	no	yes
60	no	yes
fraction	yes	yes
value greater than 480	no	yes
1,500	no	no

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. When given a description of a function in a situation, I can determine a reasonable set of inputs for the function. **1 2 3**

b. I can determine what values are possible outputs when considering the possible inputs of a function. **1 2 3**

Success looks like . . .

1. **Goal:** Given a description of a function that represents a situation, determining a reasonable set of inputs and outputs.
   - Selecting appropriate sets of numbers and values for both inputs and outputs for a verbal description of a function.
2. **Goal:** Understanding that the set of all possible outputs is dependent on the set of all possible inputs.

Suggested next steps

- If students select 25 as a possible input, consider:
  - » Having them describe what the input values represent in their own words.
  - » Reviewing Activity 1.
  - » Assigning Practice Problem 1.
- If students select "negative value" as a possible input or output, consider:
  - » Having them describe possible numbers of hours a person can sleep.
  - » Asking, "How are minutes related to hours?"

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- What worked and didn't work today? What routines enabled all students to do math in today's lesson?
- Who participated and who didn't participate in Activity 2 today? What trends do you see in participation? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: Date: Period:

1. The cost for an upcoming field trip is $30 per student. The cost of the field trip C, in dollars, is a function of the number of students x.

Select all the possible outputs for the function defined by $C(x) = 30x$.

A. 20
B. 30
C. 50
D. 90
E. 100

2. Select all the possible input-output pairs for the function $y = \sqrt{(-x)}$.

A. $(-1, 1)$
B. $(-2, 4)$
C. $(9, 3)$
D. $\left(-\frac{1}{4}, \frac{1}{2}\right)$
E. $(-16, 4)$
F. $(1, -1)$

3. A small bus charges $3.50 per person for a ride from the train station to a concert. The bus will run if at least 3 people take it, and it cannot fit more than 10 people.

Function B gives the amount of money that the bus operator earns when n people ride the bus.

a. Identify all numbers that make sense as inputs and outputs for this function.
Inputs: 3, 4, 5, 6, 7, 8, 9, 10
Outputs: $10.50, $14, $17.50, $21, $24.50, $28, $31.50, $35

b. Sketch a graph of function B.

Practice

Name: Date: Period:

4. Two functions are defined by the equations $f(x) = 5 - 0.2x$ and $g(x) = 0.2(x + 5)$. Select *all* statements that are true about the functions.

A. $f(3) > 0$
B. $f(3) > 5$
C. $g(-1) = 0.8$
D. $g(10) = f(10)$
E. $f(0) = g(0)$

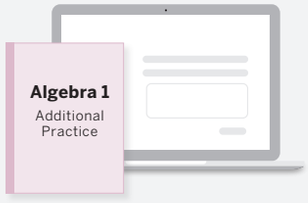
5. The graph of function f passes through the coordinate points $(0, 3)$ and $(4, 6)$. Use function notation to write each point in terms of the function f.
$f(0) = 3$, $f(4) = 6$

6. Lin built a solar-powered robot vacuum to help clean her room. The robot vacuum needs 2 hours of direct sunlight to power it for 15 minutes of vacuuming. Sketch a graph on a number line to represent x, the number of hours of direct sunlight needed to power the vacuum for at least 2 hours, but not more than 3 hours.

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 2	AI.F.3	1
	2	Activity 2	AI.F.3	2
	3	Activity 2	AI.F.3	2
Spiral	4	Unit 3 Lesson 4	AI.F.2	2
	5	Unit 3 Lesson 4	AI.F.2	2
Formative	6	Unit 3 Lesson 11	7.AF.3	2

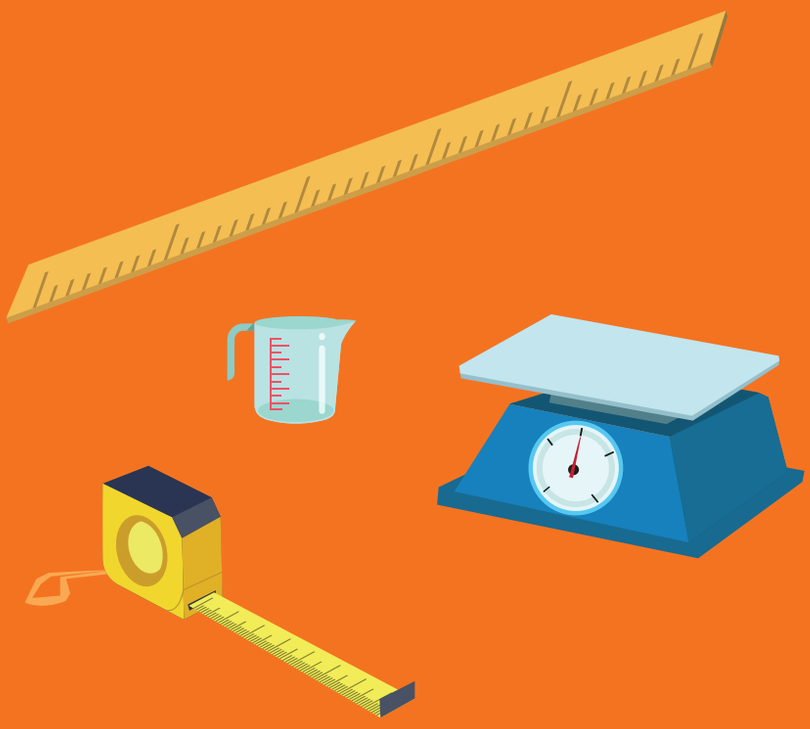
Additional Practice Available



For students who need additional practice in this lesson, assign the **Algebra 1 Additional Practice**.

My Notes:

For more information on Amplify Math, visit **indianamath.amplify.com**.



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