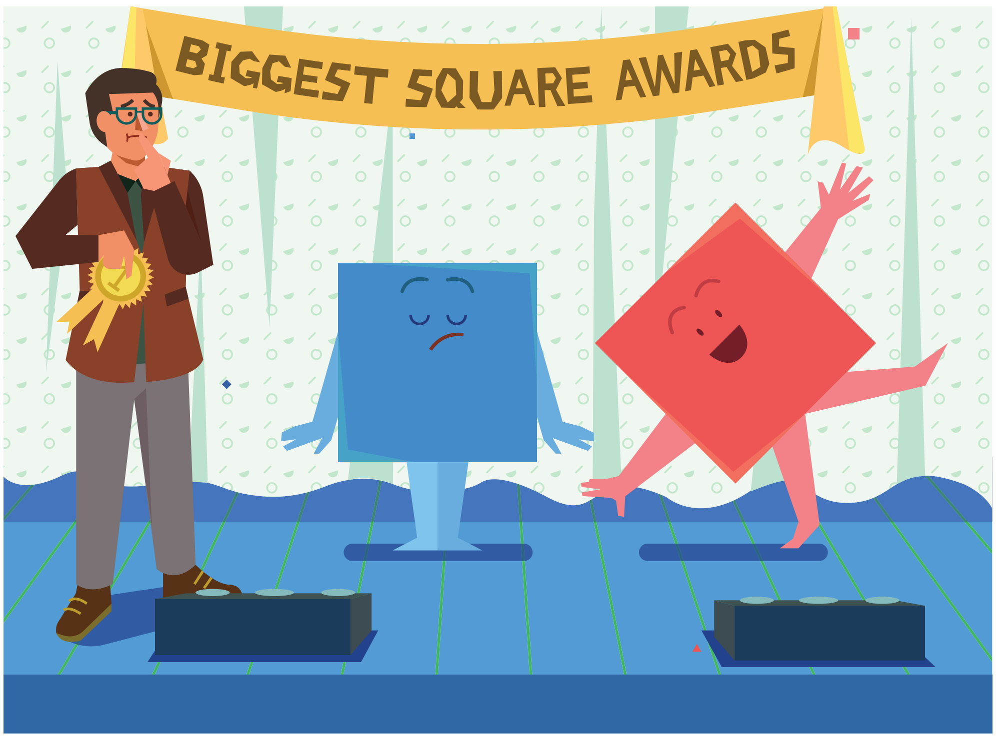
LESSON SAMPLER | TEACHER EDITION

Amplify Math
INDIANA

Grade 8

UNIT 7 | INDIANA LESSON 4A

Rational and Irrational Numbers



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Rational and Irrational Numbers

Let's explore rational and irrational numbers.

Focus

Goals

1. **Language Goal:** Understand that rational numbers are defined as numbers that can be written as fractions, representing the ratios of two integers. **(Speaking and Listening, Writing)**
2. **Language Goal:** Comprehend that numbers that are not rational are called *irrational*. **(Speaking and Listening)**

Coherence

Today

Students revisit Pythagorean thinking as they try to determine whether a fraction could represent the solution to the equation $x^2 = 2$. Students are reminded that some numbers, such as $\sqrt{2}$, are called *irrational numbers* and construct viable arguments as they critique the reasoning of others **(MP3)**.

Previously

In Grade 7, students discovered rational and irrational numbers and explored how these numbers relate to perfect squares and non-perfect squares.

Coming Soon

In Lessons 7 and 8, students will explore the relationship between repeating decimals and fractions.

Rigor

- Students build their **conceptual understanding** of irrational numbers.
- Students review irrational numbers to build **procedural skills**.

Standards

Addressing

8.NS.1

Give examples of rational and irrational numbers and explain the difference between them. Understand that every number has a decimal equivalent. For rational numbers, show that the decimal equivalent terminates or repeats, and convert a repeating decimal into a rational number.

Also Addressing: **8.NS.4**

Pacing Guide

Suggested Total Lesson Time ~**45 min**

Warm-up	Activity 1	Activity 2	Activity 3	Summary	Exit Ticket
5 min	15 min	8 min	10 min	5 min	5 min
Small Groups	Independent	Pairs	Pairs	Whole Class	Independent
		MP3	MP2		
4.NS.6*	8.NS.1, 8.NS.4	8.NS.1, 8.NS.4	8.NS.1	8.NS.1	8.NS.1

**In this activity, students build on their understanding of decimal notations for fractions from Grade 4.*

Amps powered by desmos | **Activity and Presentation Slides**

For a digitally interactive experience of this lesson, log in to Amplify Math at learning.amplify.com.

Practice

Independent

Materials

- Exit Ticket
- Additional Practice
- Activity 1 PDF, (for display)
- Anchor Chart PDF, *Perfect Squares*
- calculators (optional)

Math Language Development

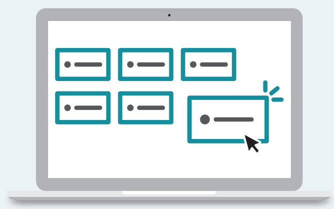
Review words

- *integer*
- *irrational number*
- *rational number*

Amps | **Featured Activity**

Activity 3
Digital Number Sort

Students sort rational and irrational numbers by dragging and connecting them on screen.



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Building Math Identity and Community

Connecting to Mathematical Practices

Students might forget that the point of a critique is to help someone, not to upset them **(MP3)**. Discuss with students what a positive critique sounds like and why they are helpful. Have students imagine being the person whose work will be critiqued, and have them describe how unkind or insensitive words would feel. Then have them describe how to communicate logic or reasoning to others in a way that would support their thinking and encourage improved arguments.

Modifications to Pacing

You may want to consider these additional modifications if you are short on time.

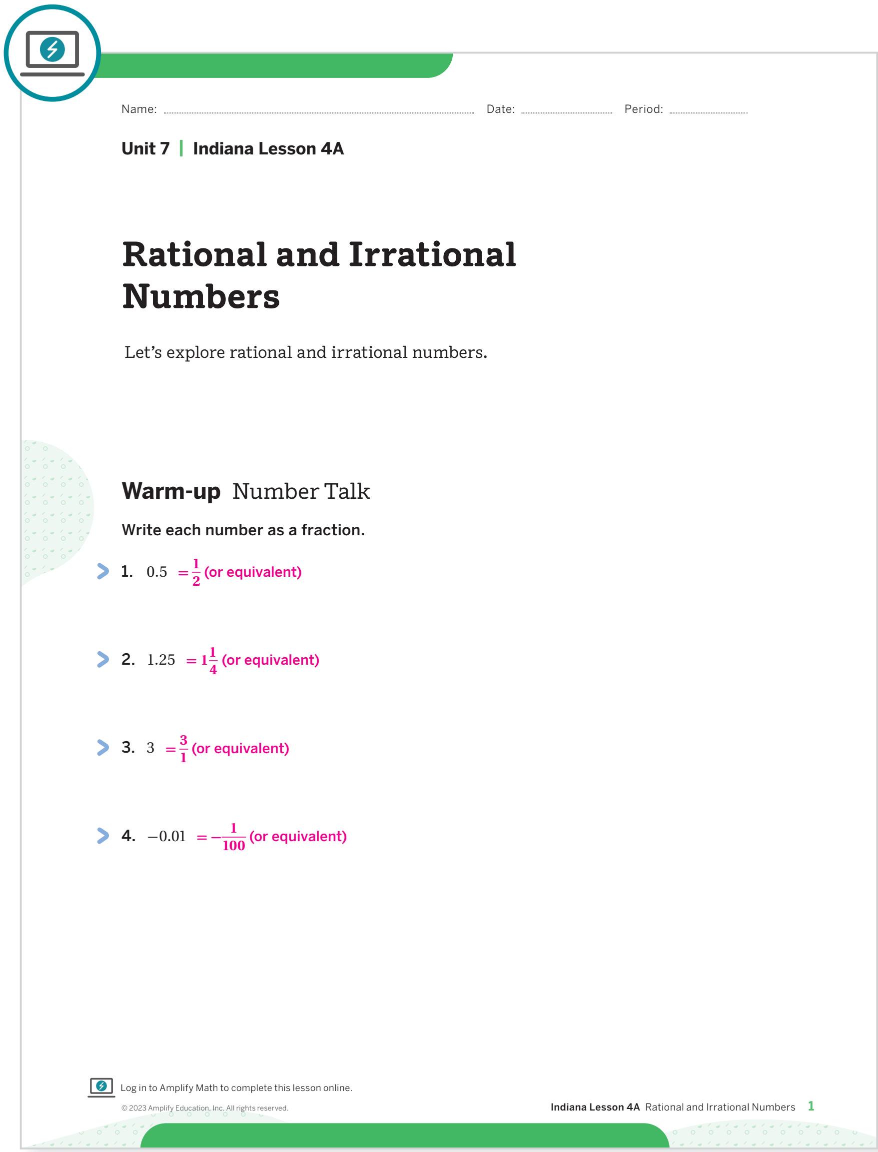
- In the **Warm-up**, Problems 2 and 4 may be omitted.
- In **Activity 1**, have students complete as many rows as time allows.
- Consider assigning **Activity 3** as practice.

Small Groups | 5 min

Warm-up Number Talk

4.NS.6

Students write decimals as equivalent fractions as a reminder that any decimal number can be written as a fraction.



1 Launch

Conduct the ***Number Talk*** routine.

2 Monitor

Help students get started by activating their prior knowledge about the relationship between place value positions and fractions.

Look for points of confusion:

- **Struggling to write 3 as a fraction.** Encourage students to look for any ratio of integers that is equivalent to 3, such as $\frac{6}{2}$ or $\frac{3}{1}$.
- **Not understanding why 3 can be written as the fraction $\frac{3}{1}$, but 0.5 cannot be written as the fraction $\frac{0.5}{1}$.** Remind students that a fraction is a ratio of integers.

Look for productive strategies:

- Writing an equivalent fraction not in simplest form.

3 Connect

Have students share their responses.

Highlight that rational numbers are numbers that can be written as a fraction, or a ratio of integers.

Ask students whether they think all numbers could be represented as a fraction to transition to Activity 1.

Power-up

To power up students' ability to evaluate expressions with an exponent, ask:

"What is the difference between $4 \cdot 3$ and 4^3?" $4 \cdot 3$ shows repeated addition, such as $4 + 4 + 4$. 4^3 shows repeated multiplication, such as $4 \cdot 4 \cdot 4$.

Use: After the Warm-up

Informed by: Performance on Lesson 4, Practice Problem 6

Independent | 15 min

Activity 1 Ratio of Integers

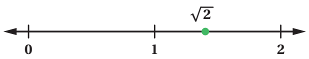
8.NS.1, 8.NS.4

Students determine whether $\sqrt{2}$ could be represented as a ratio of integers as a review of irrational numbers.

Activity 1 Ratio of Integers

Pythagoras and his followers were consumed by numbers and claimed that *all* numbers could be expressed as the ratio of integers. Let's explore whether this claim is true for $\sqrt{2}$.

Determine a ratio of integers for x that will make the equation $x^2 = 2$ true. Start by trying the fractions given in the table. Use the number line to help your thinking.



Sample response:

x	x^2
$\frac{3}{2}$	$\frac{9}{4} = 2\frac{1}{4}$
$\frac{4}{3}$	$\frac{16}{9} = 1\frac{7}{9}$
$\frac{8}{5}$	$\frac{64}{25} = 2\frac{14}{25}$
$\frac{7}{5}$	$\frac{49}{25} = 1\frac{24}{25}$
$\frac{7}{6}$	$\frac{49}{36} = 1\frac{13}{36}$
$\frac{8}{7}$	$\frac{64}{49} = 1\frac{15}{49}$
$\frac{9}{7}$	$\frac{81}{49} = 1\frac{32}{49}$
$\frac{10}{7}$	$\frac{100}{49} = 2\frac{2}{49}$
$\frac{11}{7}$	$\frac{121}{49} = 2\frac{23}{49}$
$\frac{11}{8}$	$\frac{121}{64} = 1\frac{57}{64}$

2 Unit 7 Irrationals and the Pythagorean Theorem

1 Launch

Activate students' background knowledge by asking what they remember about Pythagoras from Lesson 2. Tell students they will revisit the equation $x^2 = 2$, but, this time, they will determine whether the solution could be represented by a ratio of integers. Display the Activity 1 PDF. Model how students could use the number line to choose estimate fractions that are close to $\sqrt{2}$. Tell students that the denominator of the fraction helps them create intervals, while the numerator shows the number of intervals.

2 Monitor

Help students get started by completing the first three rows together. Have students write each improper fraction as a mixed number to see how close the number is to 2.

Look for points of confusion:

- **Struggling to determine a ratio of integers that is close to 2.** Have students choose the same denominator and adjust the numerator.

3 Connect

Have students share their closest guess for x.

Ask students whether they think there is a ratio of integers equal to x so that $x^2 = 2$.

Highlight that $\sqrt{2}$ cannot be represented as a ratio of integers. Therefore, Pythagoras's claim was not correct. Remind students that the term *irrational number* is a number that is not rational. That is, an irrational number cannot be written as a fraction representing the ratio of two integers. $\sqrt{2}$ is an example of an irrational number. Emphasize that numbers do not have to be written as fractions in order to be rational numbers.

Differentiated Support

Accessibility: *Activate Prior Knowledge*

Remind students they previously learned about rational numbers. Rational numbers can be written as a fraction, where the fraction represents the ratio of two integers. Ask students to generate examples of rational numbers.

Sample responses: $-3, -1.77, 0, \frac{2}{3}, 4\frac{5}{8}$

Extension: *Math Enrichment*

Ask students whether they agree with the statement, "Twice the square root of 2 is equal to $\sqrt{2}$." Have them explain their thinking. Sample response: I do not agree. The square root of 4 is 2 and, if twice the square root of 2 was 2, then one square root of 2 would be 1, which is not true because 1^2 does not equal 2.

Math Language Development

MLR2: Collect and Display

During the Connect, as you define the term *irrational number*, consider decomposing it to help students make sense of its definition. Add the following to the class display.

Rational	Irrational
Ratio of two integers	"Ir" + "rational" "Ir" means "not" Not the ratio of two integers

Pairs | 8 min

Activity 2 Is It Irrational?

MP3
8.NS.1, 8.NS.4

Students critique the reasoning of others to gain a better understanding of irrational numbers **(MP3).**

Name: ______ Date: ______ Period: ______

Activity 2 Is It Irrational?

Greek mathematician and Pythagorean philosopher, Hippasus of Metapontum, is credited for discovering that $\sqrt{2}$ is an *irrational number.* Hippasus' discovery of irrational numbers shocked Pythagoras because it went against his idea that all numbers could be expressed as the ratio of integers. Let's explore more irrational numbers.

1. Mai claims that any number written with a square root is an irrational number. Is Mai correct? Explain your thinking.
   Mai is not correct; Sample responses:
   - $\sqrt{16}$ is a rational number because it can be written as the ratio of integers $\frac{4}{1}$.
   - Because 16 is a perfect square, $\sqrt{16}$ is a rational number.

2. Priya, Tyler, and Clare each provide a reason why $\sqrt{108}$ is an irrational number.

   Priya: "$\sqrt{108}$ is an irrational number because 108 is a whole number that ends with an 8."

   Tyler: "$\sqrt{108}$ is an irrational number because 108 is between 100 and 121."

   Clare: "$\sqrt{108}$ is an irrational number because 108 is equivalent to $3^3 \cdot 2^2$."

   Select one of the statements and explain why the statement can be correct.
   Sample responses:
   - Priya: Perfect square numbers end in 0, 1, 4, 5, 6, or 9. 108 is a non-perfect square, so $\sqrt{108}$ is an irrational number.
   - Tyler: $10^2 = 100$ and $11^2 = 121$, so there is no whole number between 10 and 11 that makes $x^2 = 108$ true. Thus, $\sqrt{108}$ is an irrational number.
   - Clare: The product of a non-perfect square and a perfect square results in a non-perfect square, so $\sqrt{108}$ is an irrational number.

1 Launch

Have students complete Problem 1, using the *Think-Pair-Share* routine. Activate students' prior knowledge about perfect squares, rational numbers, and irrational numbers.

2 Monitor

Help students get started by reminding them that the square root of a perfect square is a rational number, while the square root of a whole non-perfect square is an irrational number.

Look for points of confusion:

- **Struggling with Problem 2.** For Priya's statement, have students list several perfect squares and ask what they notice about the ones digit. For Tyler's statement, ask students if $\sqrt{108}$ could it be written as a ratio of integers. No. Then say that $\sqrt{108}$ must be a whole number or an irrational number. Ask students if there are any additional perfect squares between 100 and 121 to support their thinking. For Clare's statement, have students determine the product of two perfect squares, and then the product of a perfect square and a non-perfect square.

3 Connect

Have students share their responses and reasoning.

Highlight that students can apply different strategies, such as the ones in Problem 2, to determine if a square root is rational and irrational.

Ask students if they think $3 \cdot \sqrt{2}$ would result in a rational or irrational number. Irrational; Because $\sqrt{2}$ is an irrational number, then three times that number is also irrational.

Note: In future grades, students will explore proofs showing why a number is irrational.

Differentiated Support

***Accessibility:** Guide Processing and Visualization, Vary Demands to Optimize Challenge*

To support students as they respond to Problem 1, provide sample numbers they could reason about as they examine Mai's claim. For example, provide the following numbers: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{9}$, and $\sqrt{10}$.

Math Language Development

MLR8: Discussion Supports

During the Connect, as you highlight the relationship between perfect squares and irrational numbers, add these statements to the class display and have students complete them. Have students brainstorm examples.

- "The square root of a whole number that is not a perfect square is ___."
  irrational (Examples: $\sqrt{2}$ and $\sqrt{6}$)

Pairs | 10 min

Activity 3 Is It Rational or Irrational?

MP2
8.NS.1

Students classify a number as rational or irrational to build procedural fluency.

Amps Featured Activity Digital Number Sort

Activity 3 Is It Rational or Irrational?

1. Determine whether each number is rational or irrational. If the number is rational, write R. If the number is irrational, write I.

5 R	-2.051 R	$\sqrt{10}$ I	$\sqrt{16}$ R
$\sqrt{13}$ I	$\sqrt{1{,}098}$ I	$2\frac{1}{4}$ R	3.1 R
$\sqrt{1}$ R	$\sqrt{\frac{9}{16}}$ R	$\frac{8}{2}$ R	$-\sqrt{2}$ I

2. Choose one rational number and one irrational number from Problem 1 and explain how you know the number is rational or irrational.
Sample response: $\sqrt{16}$ **is rational because 16 is a perfect square.** $\sqrt{13}$ **is irrational because 13 is not a perfect square.**

Are you ready for more?

Determine whether the expression $1 + \sqrt{2}$ will result in a rational or irrational number. Explain your thinking.

Irrational; Sample response: Because $\sqrt{2}$ **is irrational, a whole number added to an irrational number will result in an irrational number.**

STOP

4 Unit 7 Irrationals and the Pythagorean Theorem

1 Launch

Set an expectation for the amount of time students will have to work in pairs on the activity.

2 Monitor

Help students get started by having them write each number as a ratio of integers, if it is possible.

Look for points of confusion:

- **Trying to determine if a number is rational by looking for an equivalent fraction.** Have students focus on identifying irrational numbers first by looking for any square roots that do not produce a rational number.
- **Mislabeling $-\sqrt{2}$ as rational.** Have students refer to Activity 1 where they saw that $\sqrt{2}$ is irrational. Ask them to reason abstractly or quantitatively to determine why $-\sqrt{2}$ is irrational **(MP2)**.

3 Connect

Have students share their responses, discussing any discrepancies.

Ask students to explain why they think $-\sqrt{2}$ is a rational or irrational number. Explain that if $\sqrt{2}$ cannot be written as a rational number, $\frac{a}{b}$, then $-\sqrt{2}$ cannot be written as $-\frac{a}{b}$.

Highlight that when looking for irrational numbers, students can focus on square roots. They can look for square roots that do not produce a rational number, and identify these numbers as irrational. Remind students that, while these are not the only irrational numbers, using this method may help them determine whether a number is rational or irrational.

Differentiated Support

Accessibility: *Optimize Access to Technology*

Have students use the Amps slides for this activity, in which they can sort rational and irrational numbers by dragging and connecting them on screen.

Accessibility: *Activate Prior Knowledge, Guide Processing and Visualization*

Provide students with copies of the Anchor Chart PDs, *Perfect Squares*.

Math Language Development

MLR3: Critique, Correct, Clarify

During the Connect, present and incorrect statement that reflects a possible point of confusion from the class. For example, "$\sqrt{10}$ is rational because 10 can be written as the fraction $\frac{10}{1}$. Ask:

- ***Critique:*** "Do you agree or disagree with this statement? Explain your thinking." Listen for students who reason that while 10 is rational, $\sqrt{10}$ is not rational because 10 is not a perfect square.
- ***Correct:*** "Write a corrected statement."
- ***Clarify:*** "How can you convince someone that your statement is correct? What mathematical language or reasoning can you use?"

English Learners

Have students correct the statement by first writing "$\sqrt{10}$ is irrational because 10 is/is not a perfect ___."

Whole Class | 5 min

Summary

8.NS.1

Review and synthesize rational and irrational numbers.

Name: ______ Date: ______ Period: ______

Summary

In today's lesson . . .

You explored rational and irrational numbers. A *rational number* is a number that can be expressed as a fraction, in other words, as a ratio of two integers. An *irrational number* cannot be expressed as a fraction.

Examples of rational numbers	Examples of irrational numbers
$\frac{7}{4}$, 0, 0.2, $-\frac{1}{3}$, $\sqrt{9}$	$\sqrt{2}$, $\sqrt{27}$, π

Reflection:

Synthesize

Have students share how they know whether a number is rational or irrational in their own words.

Ask students to provide an example of a rational number and an irrational number.

Highlight that rational numbers can be expressed as fractions, while irrational numbers cannot be expressed as fractions. Also highlight that the square root of a whole number, perfect square is rational, while the square root of whole number, non-perfect square is irrational.

Reflect

After synthesizing the concepts of the lesson, allow students a few moments for reflection on one of the Essential Questions for this unit. Encourage them to record any notes in the *Reflection* space provided in the Student Edition. To help them engage in meaningful reflection, consider asking:

- "What is the difference between a rational number and an irrational number?"

Independent | 5 min

Exit Ticket

8.NS.1

Students demonstrate their understanding by describing a rational and irrational number and providing examples of each.

Printable

Name: ____________ Date: ________ Period: ________

Exit Ticket 7.4A

1. In your own words, describe a *rational number*. Give at least three different examples of rational numbers.
   Sample response: A rational number can be written as a fraction. For example, $\frac{1}{2}$, 0.3, and $\sqrt{25}$.

2. In your own words, describe an *irrational number*. Give at least three different examples of irrational numbers.
   Sample response: An irrational number cannot be written as a fraction. For example, $\sqrt{2}$, $\sqrt{29}$, and π.

Self-Assess

1 I don't really get it | 2 I'm starting to get it | 3 I got it

a. I know what a rational number is and can give an example. 1 2 3

b. I know what an irrational number is and can give an example. 1 2 3

Success looks like . . .

1. **Language Goal:** Understanding that rational numbers are defined as numbers that can be written as a fraction, representing the ratios of two integers. **(Speaking and Listening, Writing)**
   - Describing rational numbers as fractions, ratios of integers, or the square root of perfect squares in Problem 1.
   - Writing three rational numbers in Problem 1.
2. **Language Goal:** Comprehending that numbers that are not rational are called *irrational*. **(Speaking and Listening)**
   - Describing an irrational number as numbers that cannot be represented as the ratios of two integers or as the square root of perfect squares in Problem 2.
   - Writing three irrational numbers in Problem 2.

Suggested next steps

- **If students do not correctly describe a rational or irrational number, but provide correct examples, consider** reviewing the definitions of a rational number and irrational number.
- **If students do not provide correct examples of rational numbers or irrational numbers, consider** reviewing Activity 3.

Professional Learning

This professional learning moment is designed to be completed independently or collaboratively with your fellow mathematics educators. Prompts are provided so that you can reflect on this lesson before moving on to the next lesson.

Points to Ponder . . .

- The focus of this lesson was for students to learn about irrational numbers. How well do you think your students understand the concept of irrational numbers and are they able to distinguish them from rational numbers?
- Which groups of students did and didn't have their ideas seen and heard today? What might you change for the next time you teach this lesson?

Practice

Independent

Practice

Name: ______ Date: ______ Period: ______

1. Write each number as a fraction. If it is impossible, write *irrational number*.
   - a. 0.01 $= \frac{1}{100}$
   - b. $-\sqrt{144}$ $= -\frac{12}{1}$
   - c. $\sqrt{37}$ Irrational number
   - d. $\sqrt{25}$ $= \frac{5}{1}$
   - e. $\sqrt{12}$ Irrational number

2. Noah says that the solution to equation $x^2 = 120$ is rational. Do you agree or disagree with Noah? Explain your thinking.
   Sample response: I disagree with Noah. The solution is $x = \sqrt{120}$ and 120 is not a perfect square, so the solution is an irrational number.

3. Determine whether each statement is true or false. Explain your thinking.
   - a. The sum of two positive whole numbers will always result in a rational whole number.
     True; Sample response: The sum of two whole numbers is a whole number and all whole numbers are rational.
   - b. The sum of two positive rational numbers, that are not whole numbers, is never a whole number.
     False; Sample response: $\frac{1}{2} + \frac{3}{2} = 2$
   - c. The sum of two rational numbers is always a rational number.
     True; Sample response: The sum of two fractions can always be written as a fraction.

Name: ______ Date: ______ Period: ______

4. Determine which expression is larger, and then estimate how many times larger.
   Sample responses shown.
   - a. 0.37×10^6 and 700×10^4
     700×10^4 is about 19 times larger;
     $\frac{700 \times 10^4}{0.37 \times 10^6} = \frac{700 \times 10^4}{37 \times 10^4} \approx \frac{700}{37} \approx 19$
   - b. $500{,}000$ and 2.3×10^8
     2.3×10^8 is about 460 times larger;
     $\frac{2.3 \times 10^8}{500000} = \frac{2.3 \times 10^8}{0.005 \times 10^8} \approx \frac{2.3}{0.005} = 460$

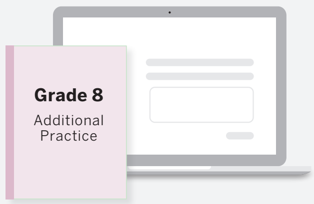
5. Han makes a trip to Mexico. He exchanges some dollars for pesos at a rate of 20 pesos per dollar. While in Mexico, he spends 9,000 pesos. When he returns, he exchanges his pesos for dollars (still at 20 pesos per dollar). He receives $\frac{1}{10}$ the amount he started with. Determine how many dollars Han exchanged for pesos at the beginning of this trip and explain your thinking.
   $500
   Sample response: Let x represent the number of dollars that Han exchanged at the beginning of his trip.
   $$\frac{20x - 9000}{20} = \frac{1}{10}x$$
   $$20 \cdot \frac{20x - 9000}{20} = \frac{1}{10}x \cdot 20$$
   $$20x - 9000 = 2x$$
   $$-9000 = -18x$$
   $$500 = x$$

6. Use long division to write $\frac{3}{16}$ as a decimal. Show your thinking.
```
     0.1875
16)3.0000
  -16
   140
  -128
    120
   -112
      80
     -80
       0
```
   $\frac{3}{16} = 0.1875$

Practice Problem Analysis

Type	Problem	Refer to	Standard(s)	DOK
On-lesson	1	Activity 3	8.NS.1	1
	2	Activity 1	8.NS.1, 8.NS.4	2
	3	Activity 2	8.NS.1	3
Spiral	4	Unit 6 Lesson 13	8.C.2	2
	5	Unit 4 Lesson 6	8.AF.1	2
Formative	6	Unit 7 Lesson 7	6.NS.5	1

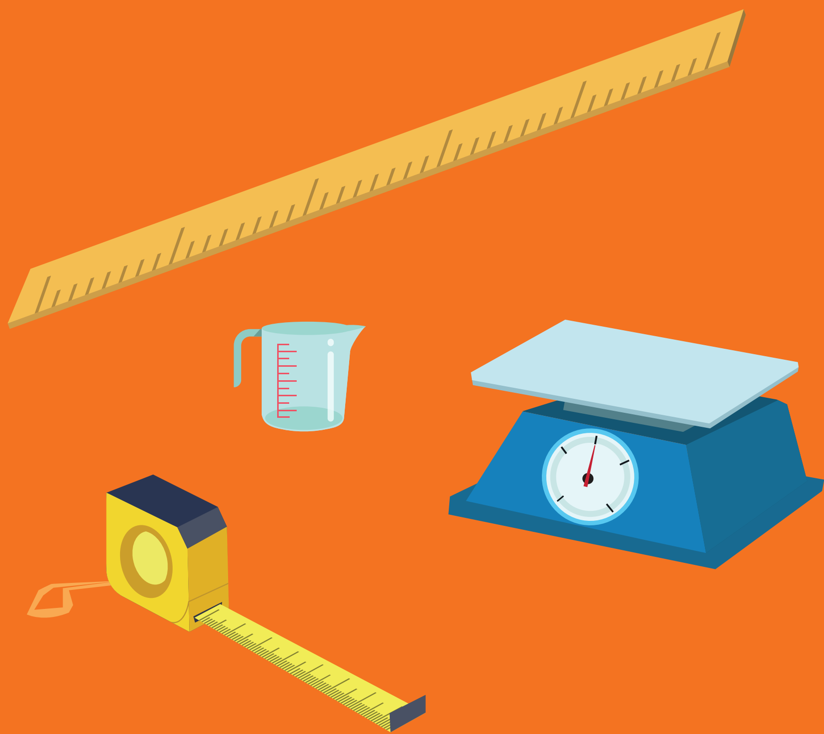
Additional Practice Available



For students who need additional practice in this lesson, assign the **Grade 8 Additional Practice**.

My Notes:

For more information on Amplify Math, visit **indianamath.amplify.com**.



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